V.—*Account of the Verification of some Standard Weights with considerations on Standard Weights in general.—By* Col. J. F. Tennant, R. E., F. R. S., *Master of Her Majesty's Mint.*

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When I first contemplated the verification of a series of weights from a primary standard, I had little information as to procedure, and indeed I have till now had little as to details. I had intended in this paper to deal with the verification of a whole series of ounce weights; but circumstances beyond my control have delayed the latter portion, and I think that probably this shorter paper will be as much as the patience of my readers will stand: in it are described, with examples, all the cases I shall meet; while the explanations will, I trust, enable any one to follow my procedure and somehow to verify any other set of weights. This end being gained, the delay of the paper to add the numerical results of farther work, would add little to its popular, or even scientific value, and this circumstance has induced me to offer it in its present state to the Asiatic Society.

I am aware that I am open to the charge of excessive (factitious) accuracy, and I freely admit that I have used an excessive number of decimal places; but the number was originally fixed by the fact that it caused no trouble and saved thought. The difference between the trouble of dealing with 5 or 6 figures and 4 with an arithmometer is, in my case, more than compensated by the absence of the absolute necessity of watching the increase of the last figure: and too, I had not, till I had gone some way with these weighings, so clear an idea of the probable errors as I now have. The systematic calculation of these is, so far as I know, new: it has taught me much, and guided me where I might have gone wrong. I think that it should always be carried out; but of course, the foundation of the calculation—the estimation of the probable error of one comparison, will not commend itself to all men:—those who in other respects may follow my procedure may prefer a different course in this, and, when the system of weighment is different, this datum must be determined in a correspondingly different manner. Even then, I hope, that the conclusions I have come to may have their use, for the evidence they offer of the rapid accumulation of error in multiplying from a small primary standard, is quite independent of the amount ascribed to the error of one comparison.

I have added the Tables requisite in reducing the comparison of weights of varying density and in determining specific gravity. These are deduced from the same data precisely as those used in the British Standards Department, but I have employed Fahrenheit's thermometer, the English inch, and the English grain, because, to me, those units were more accessible (as they will be to most readers of the English language) and not because I prefer them. I have thought that it was more important to avoid conversions of the data before using them than to adhere to general considerations; just as (with the late Warden of the Standards) I have preferred uniformity of data for reduction; rather than a possible scientific accuracy, which is, after all, not demonstrably gained.

Section I.—*On Weights.*

In May 1879, I received from England a set of Bullion Weights of gilt bronze, with their errors on the Commercial Standard of England roughly given, and a Troy Ounce of Platinum-iridium, with its error in vacuo in terms of the Parliamentary Standard Pound P S. I at the same time received a set of Metric Weights of Platinum-iridium from 100 grammes to one milligram, with their errors in terms of the Kilogramme des Archives, which is the Normal Standard weight of France. My paper here will be confined to dealing with some of the Bullion Weights: and it will be necessary in order to understand the procedure I follow, and also the scientific principles of weighing, that I should give an account of the English system of weights.

Ordinary weights are made of brass, iron, or some other cheap metal, but all these are liable to oxidation, and thus none of these metals is suitable for a Standard. The metal chosen for the English Standard was platinum, which is nearly indestructible. Since then it has been found that, whereas platinum is soft, an alloy with iridium is hard, has the other advantages of platinum, and can be made with sufficient readiness for the purpose required: this alloy is used in my Primary Standards as it is in the European Standards now being made in Paris. The use of such substances for Standard Weights, however, leads to some complication: these metals are heavy; while the metals and alloys ordinarily used are comparatively light. Now the weight of a body in air is different from its weight in vacuo by the weight of the air displaced, and this varies with the state of the atmosphere: consequently the relative weight of a pound of brass and one of platinum, which are alike in vacuo, will, in air, be found to vary continually relatively to each other. In order to avoid the inconvenience of this, it has been found desirable that the Commercial Standard should be of brass or bronze; both of which, having nearly the same density as the metals used in ordinary weights, will show the same differences at all times and places, with sufficient accuracy for commercial purposes; and which, moreover, are cheap enough to allow of the weights of all sizes being made of them. For general Standard purposes, weights are now made of gilt bronze, the gilding preserving them to a great extent from changing by oxidation.

As the Parliamentary Standard of England P S. has its true weight in vacuo,* the first impression would be, that the Commercial Standard in ordinary air should weigh the same as P S. in vacuo : but this has not been the practical solution. When the Houses of Parliament were destroyed in 1834, the English standards were destroyed in them, and the new Standard was meant to be a restoration of the old one. Now the old Standard was a brass Troy Pound made in 1758, of which there were a variety of copies more or less accurate. On the evidence from these, and some other sources, was determined the difference between the lost pound and a piece of platinum, both taken in vacuo. Then (the Government of the day having determined that the new Standard should represent the Avoirdupois, and not the Troy Pound as before), a second piece of Platinum P S. was made which should weigh very nearly 7000 such grains as those of which the destroyed Pound (U) contained 5,760, both being taken in vacuo, and it is believed that the result was accurate to a very small fraction of a grain, thanks to the great labours of Professor Miller. In reverting to the Commercial Pound, that would be 7,000 grains of which U weighed 5,760, both taken in air, and then, as the density of the new commercial Pound was very close to that of U, all sensible uncertainty arising from the destruction of U and the impossibility of knowing its exact density would vanish.

Professor Miller found the Platinum Pound P S. to be 7000·00093 grains of U both weighed in vacuo, and by Act of Parliament, *this* was declared to be the true standard of weight, and that one grain should be a seven-thousandth part of it. The Commercial Pound W was an imaginary Pound, supposed to be made of brass of a density of 8·15034, which was what Professor Miller estimated as the density of the lost Pound U. Though the standard in vacuo was changed, as above, by a minute quantity, it would have been wrong to change the weight of W in air. In order then that its weight in vacuo should become that of the Pound P S., it became necessary to suppose that this weight in vacuo†, and consequently its density, were changed, and to ascribe to it a new density of 8·1430.

The present definition of the English Commercial Pound then is—

* I have followed the wording of my predecessors, but I should prefer to call the "*weight in vacuo*" the "*Mass*," and restrict the term "*weight*" to the apparent force exercised. If this distinction were made, the questions involved would be much clearer. The Parliamentary Standard has been treated as one of *Mass ;* hence two of the gilt secondary standards, each of the same Mass as P. S., will not have ordinarily the same *weight*, unless they have the same specific gravity.

† The weight in vacuo was 7000 grains of U, and in consequence of the Act of Parliament it became necessary that it should be the same as that of P S. or 7000·00093 grains of U.

The weight in standard air of a piece of brass whose weight in vacuo is the same as that of P S., and whose density, compared with that of water at its maximum density (the brass being at the freezing point), is 8·1430.

If we know the value of a weight in terms of P S, we shall be able to find its value in terms of W by adding the weight of air displaced by the same weight of brass similar to that of which W is supposed to be made, and deducting that actually displaced by the weight to be determined.

The Standard Platinum-Iridium ounce sent me is certified to weigh (in vacuo) 479·95979 grains in terms of P S., and the density has been assumed as 21·414, which is that of the 100 gramme weight. In English Standard Air its weight is given as 480·00502 grains, but that datum is useless for purposes of reference. It is called E I in the books of the Standards Office in London, and I propose to retain this name.

The ounce weight of the bullion set was certified to weigh 480·00145 grains in vacuo in terms of P S. and 480·00203 grains in English Standard Air in terms of W.

The following matter must be borne in mind in order that the procedure in my weighments may be understood:

The sign $=$ means that the weights on each side of it are equal in vacuo.

The sign $\equiv$ means that these are equal in air at the time; and, in the case of Commercial Weights, that they are sufficiently equal for practical purposes at all times.

The sign $\triangleq$ means that the weights on each side being in the respective pans of the balance there would be equilibrium. When no division of the scale is mentioned as the resting point, it is assumed to be 10 for Oertling No. 1 and 15 for Oertling No. 2.

O_n is one of the set of Gilt Bullion Weights—the subscript number denotes its nominal value in Troy ounces.

P_n is one of a set of grain weights which have been used for small quantities, and n is the number of grains nominally: all weights not less than 1 grain are of platinum and have been cleaned by incandescence in a spirit-lamp. The tenths of grains are of aluminum and the hundreths of uncertain material.

R_1 and R_2 are two riders (approxinately of one-tenth of a grain each) used with the balance Oertling No. 1.

The Tables I have used in my reductions have been calculated by myself to the units of the Barometer and Thermometer scales commonly used in England, and which it was most easy for me to refer to. That for the density of air, has been calculated from the formula given by Professor Miller, in his paper in the Philosophical Transactions, with the necessary changes for units, and for the position of Her Majesty's Mint at Calcutta. The density of water has been calculated from a formula similar to Professor Miller's; but with the constants deduced from the new Tables of the British Standards Office. The other Tables, for the expansion of metals, are deduced from the same data as those of Professor Miller, but the form makes them more compact and convenient without any loss of accuracy. All will be found at the end.

Section II.—*The Balances.*

Oertling No. 1 is a chemical balance by Oertling with a beam 365 m. m. (14·56 inches) between the extreme knife edges. The principal knife edge is 28 m. m. (1·1 inches) long and the smaller ones 16·5 m. m. or 0·65 inches; all are of agate resting on agate planes. The beam is divided for the use of riders, and I have satisfied myself that the divisions are sufficiently accurate for this purpose. The scale is placed on the lower part of the pillar, and is read by a long index attached to the centre of the beam: this is in my opinion, the best arrangement.

Oertling No. 2 is a balance whose beam carries knife edges 404 m. m (15·9 inches) apart. The central knife edge is 38·4 m. m ($1\frac{1}{2}$ inches) long and those at the ends, 22 m. m or 0·87 inches. They are all of agate and rest on agate planes. The beam is very strong, and divided with sufficient accuracy for the use of a rider. There is an index of soft iron at each end of the beam to read an ivory scale. The left scale had very fine graduations and appeared to me useless. I have substituted a better one and removed the right scale.

Section III.—*Density of O Set of Weights.*

In order to compare O_1 with EI it is necessary to have a density of O_1: I have determined that of O_3 and assumed it to be the same as that of O_1 and of the other O weights.

It appears from the papers received from the Standards Office that $O_3 \equiv 3$ Troy ounces $\equiv$ 1440 grains with sufficient accuracy for this purpose, its exact value will be seen later.

On July 4th 1879, the balance Oertling No. 1 having been prepared for taking specific gravities, and a platinum hook, intended to support O_3 in water, having been hung by a fine wire of platinum so as to be immersed in distilled water; O_3 was placed in the pan in air, and counterbalanced with weights. O_3 being then placed in the hook, and all air bubbles carefully removed, it was found that; X being about 1490·2 grains:

$X \triangleq O_3$ in water (temp. $= 84°.\ 1$) + hook &c. in water + ($O_{\cdot 3}$ + $O_{\cdot 04}$ + $O_{\cdot 005}$ + $O_{\cdot 004}$) in air + $4.\ \frac{R_2}{10}$ at 10·02 divisions of the scale—

then, removing O_3 from water, carefully drying it, and placing it in the pan, I found after adding 180 minims of water

$$X \bumpeq O_3 \text{ in air} + \text{hook \&c. in water} + 2{\cdot}72\,\frac{R_2}{10} \text{ at } 10{\cdot}02 \text{ divisions.}$$

Hence the loss of weight apparently $= O_{\cdot 3} + O_{\cdot 04} + O_{\cdot 005} + O_{\cdot 004} + 1{\cdot}28\,\frac{R_2}{10}$.

My approximate calculations gave me the sum of the above four weights as 167·5400 grains, and the value of the rider is approximately $\frac{1}{10}$th of a grain, the difference from the true value being negligible. Hence the loss of weight between air and water was 167·5528 grains, and, though I did not observe the Barometer, it may be considered as 29·46, and the temperature 87°·5; this gives $\Delta\, O_3 = 8{\cdot}5649$.

Again on July 7th, I found in the same way.

$$\text{(A)}\quad X + 5\,\frac{R_1}{10} \bumpeq O_3 \text{ in water} + \text{hook \&c. in water} + 167{\cdot}54 \text{ grains} + 3\,\frac{R_2}{10} \text{ at } 13{\cdot}30 \text{ Div.}$$

$$\text{(B)}\quad X + 5\,\frac{R_1}{10} \bumpeq O_3 \text{ in water} + \text{hook \&c. in water} + 167{\cdot}54 \text{ grains} + 6\,\frac{R_2}{10} \text{ at } 4{\cdot}72 \text{ Div.}$$

Temp. water 84·25 F.

and, after adding 169 minims of water.

$$\text{(C)}\quad X + 5\,\frac{R_1}{10} \bumpeq O_3 \text{ in air} + \text{hook \&c. in water} + 7\,\frac{R_2}{10} \text{ at } 14{\cdot}80 \text{ Div.}$$

Bar. 29°·445.

$$\text{(D)}\quad X + 5\,\frac{R_1}{10} \bumpeq O_3 \text{ in air} + \text{hook \&c. in water} + 9\,\frac{R_2}{10} \text{ at } 8{\cdot}35 \text{ Div.}$$

Temp. 85°·7 F.

Hence by interpolating between (A) and (B)

$$X + 5\,\frac{R_1}{10} \bumpeq O_3 \text{ in water} + \text{hook \&c. in water} + 167{\cdot}54 \text{ grs.} + 4{\cdot}14\,\frac{R_2}{10}$$

and from (C) and (D)

$$X + 5\,\frac{R_1}{10} \bumpeq O_3 + \text{hook \&c. in water} + 8{\cdot}49\,\frac{R_2}{10}$$

Temperatures Water 84°·25F. Air 85·7
Bar. 29·445 in.

Thus the loss of weight was apparently 167·4965 grains, and $\Delta\, O_3 = 8{\cdot}5676$. Giving this last result triple weight, on account of better observing, we have as a mean; $\Delta\, O_3 = 8{\cdot}5669$: which may be considered the density for all the weights of this set; and which will not be altered by the true values of the weights used, being substituted for the approximate ones.

Section IV.—*System of Weighments.*

I have adopted a uniform system of weighment for comparing the weights. Some years ago I made a considerable number of experiments on the species of errors which occurred in practice, and the present system is the outcome: there have been minute deviations, but in all material points the procedure has been uniformly followed, and I think it has been successful in eliminating all progressive errors. The principal of these is the tendency of the arms of the balance to expand unequally with temperature, but there are others which have occasionally been found. I annex specimens of the form I have used in work.

The weights to be compared being placed in the pans, a preponderance is given to one side of the balance; so as to make the resting point, when the whole is in equilibrium, lie on one side of the centre point; yet so slightly, that the weight used to get the value of the scale, shall deflect the resting point to the other side. In the first example with Oertling No. 1, it will be seen, that with EI in the left pan and O_1 in the right, the Right Rider was placed at 1·2 of the beam scale; in this state the index had its resting point at 7·54 divisions (10 being the middle). Then the weight $P_{\cdot 01}$ was added to the left side and the resting point became 15·81 Div. Each resting point is deduced from 4 readings, two low l_1 and l_2, and two high h_1 and h_2. The beam having been carefully released, the first excursion outwards, and the return towards the scale centre, are neglected; and the next four readings of the extremes of oscillation taken. The first reading will thus usually be low, if the resting point be low; and high, if that be high: but, when signs of irregularity occur, this may not be the case, as I have always, in such cases, freely omitted readings till the oscillations have become regular. Then, supposing a low reading first, $\frac{l_1 + 2h_1 + l_2}{4}$ and $\frac{h_1 + 2l_2 + h_2}{4}$ would be readings of the resting points, and the sums in the numerators have been rapidly formed separately during the work, added, and divided by 8. This has been afterwards checked by $\frac{l_1 + h_2 + 3\ (l_2 + h_1)}{8}$: of course, when h comes first, the h's take the place of the l's in these formulæ, and *vice versâ.*

We thus have two "*partial weighments*"

$$\text{EI} \simeq O_1 + 1{\cdot}2\,\frac{R_2}{10} \text{ at } 7{\cdot}54 \text{ divisions and}$$

$$\text{EI} + P_{\cdot 01} \simeq O_1 + 1{\cdot}2\,\frac{R_2}{10} \text{ at } 15{\cdot}81 \text{ divisions}$$

from which I get, by interpolation, as a result of the "*weighment*"

$$EI \doteq O_1 + 1{\cdot}2\,\frac{R_2}{10} - P_{\cdot 01}\cdot\frac{2{\cdot}46}{8{\cdot}27} \text{ or } O_1 + 1{\cdot}2\,\frac{R_2}{10} - 0{\cdot}297\,P_{\cdot 01}$$

The second weighment is made after the weights are interchanged in the pans and the result deduced the same way. These together make one "*comparison;*" and then a second comparison is made, every operation being followed, but precisely in the reverse order, to make a "*complete comparison.*" The result of the four equations when summed is

$$4\,EI \equiv 4\,O_1 + 0{\cdot}191\,P_{\cdot 01} \text{ or}$$
$$EI \equiv O_1 + 0{\cdot}04775\,P_{\cdot 01}$$

The interpolations are made with sufficient accuracy with a slide rule.

In all the comparisons of the O set and P set, except those of EI with O_1, which were made with the balance Oertling No. 1, I have used one of the riders (the right) to add a constant weight to one side and the other in variable positions. Assuming that the rider can be accurately placed on the divisions, and that these are sufficiently accurate, it seems to me that I may safely use the rider in this way, and that the error of determination of the weight of the rider will thus be of less importance than that of a small weight.

In the case of the very small weights I have added the weight P_{24} to one pan, and P^{*}_{24} to the other, in order to steady them, with great advantage.

Section V.—*Determination of O_1, in terms of the English Commercial Pound*

I have before mentioned that I have received as a Standard a Troy ounce of Platinum-Iridium, whose weight in terms of the Parliamentary Standard Pound P S. is 479·95979 grains of P S.; and I have explained the relations between the English Standard Pound and the commercial Pound. In order that I may determine the errors of the Bullion set of Weights, it is necessary that I should determine O_1 in terms of the English Commercial Pound: I have it is true the determination made in London, but it is necessary to verify this, not only to make the standard of weight now, identical with that I should get again, but also because the gilt weights may have slightly changed in the long voyage.

The Barometer I have used is an Aneroid Barometer by Browning, which I have found give corrected Barometer readings without sensible error. I have, except in the first comparison, used two Thermometers which were examined for me some years ago at Kew, and whose zero point I have recently re-determined: these were suspended in the balance case of Oertling No. 1, so as to hang about half way between the pillar carrying the central plane, and the suspensions of the scale pans. The Humidity has been deduced from a new Masons Hygrometer: I have not the errors of its Thermometers, but they are modern, and not likely to have any producing sensible corrections to my result.

The following is a specimen of computation for the comparison of EI and O_1 which is entered in the type form; in it, v EI = volume of water at its greatest density which is displaced by EI at 32°. F.

$$\text{it therefore} = \frac{wt.\ \text{EI}}{\Delta\text{EI}} = \frac{479{\cdot}95979}{21{\cdot}414} = [1{\cdot}35051]$$

$$\text{similarly } v\ O_1 = \frac{479{\cdot}99760}{8{\cdot}5669} = [1{\cdot}74842]$$

May 24th, 1879 A. M.

Commenced at 6 h. 48 m. Ended at 7 h. 33 m.

Dry Bulb 85°·9 F.	Dry Bulb 85°·4
Wet do. 81·0 } Vapour Tension	Wet do. 80·1 } Vapour Tension
Diff. 4·9 } 0·993 in.	5·3 } 0·960 in.

Mean of Thermometers 85°·5 Mean Red. Barometer 29·605 in.

Correction 0·00 0·189(0·993 + 0·960) = 0·369

Mean Temperature 85·50 h. = 29·236 log — 1·46592

log A_t (Tab I.) 5·59005

	7·05597	....	7·05597
log v EI	1·35051	log v O_1 ...	1·74842
(Tab. III.) log (1 + E P_t)	0·00035	(Tab. III.) log (1 + E B_t) ...	0·00066
Air displaced by EI = 0·025517 grs. } log =	8·40683	Air displaced by O_1 = 0·063834 grs. } log =	8·80505

		grains.
Weight EI in Vacuo	=	479·95979 of P S.
Air displaced	=	— 0·025517
EI	≡	479·934273
Air displaced by O_1	= +	0·063834
O_1 ≡ EI	—	0·000475*
O_1	=	479·997632

* In section IV, I found EI = O_1 + 0·4775 $P_{\cdot01}$ and (Sec. VI) $P_{\cdot01}$ = 0·009947 grains.

Abstract of Comparisons.

1879 May 24,	O_1	=	479·997632	P S. grain.
„ 28,	„		·997489	„
„ 30,	„		·996732	„
„ 31,	„		·997266	„
„ June 1,	„		·996911	„

Mean $O_1 = 479·997206 \pm 0·000115$ P S. grains.

I have received, from the Meteorological Reporter to the Government of Bengal, the following mean data for Calcutta which I take as the definition of Standard Air

	in.	
Reduced Barometer,...	29·787	whence $\Delta A_s = 7.06510$.
Temperature,............	79°·0 F.	
Humidity,...............	0·76 per cent.	

Hence I have weight of O_1 =	479·997206	grains of P S.
Deduct displaced Standard Air =	— 0·065178	
Add Standard Air for $\frac{480}{7000}$ W =	+ 0·068571	
O_1 ≡	480·000599	grains of English Commercial Pound.

This value differs slightly from that sent me and which I have quoted before.

SECTION VI.—*On the determination of the errors of single weights.*

In the interval between O_1 and O_{10} there are, in all English bullion sets, weights O_5, O_4, O_3, and O_2; so between O_{10} and O_{100} come O_{20} O_{30} O_{40} and O_{50}, and so on.

Between these weights we may make comparisons giving the following equations:

$$\begin{aligned}
O_{10} &\equiv O_5 + O_4 + O_1 + x_1 \pm e \ (a) \\
&\equiv O_5 + O_3 + O_2 + x'_1 \pm e \ (b) \\
&\equiv O_4 + O_3 + O_2 + O_1 + x''_1 \pm e \ (c) \\
O_5 &\equiv O_4 + O_1 \qquad + x_2 \pm e \\
O_5 &\equiv O_3 + O_2 \qquad + x_3 \pm e \\
O_4 &= O_3 + O_1 \qquad + x_4 \pm e \\
O_3 &= O_2 + O_1 \qquad + x_5 \pm e
\end{aligned}$$

e being the *p. e.* of one comparison.

Hence we have $O_2 \equiv 2\,O_1 + x_4 - x_3 + x_2 \pm e\sqrt{3}$

$$O_3 \equiv 3\,O_1 + x_5 + x_4 - x_3 + x_2 \pm e\sqrt{4}$$

$$O_4 \equiv 4\,O_1 + x_5 + 2x_4 - x_3 + x_2 \pm e\sqrt{7}$$

$$O_5 \equiv 5\,O_1 + x_5 + 2x_4 - x_3 + 2x_2 \pm e\sqrt{10}$$

$$O_{10}\begin{cases}\equiv 10\ O_1 + 2x_5 + 4x_4 - 2x_3 + 3x_2 + x_1 \pm e\sqrt{34} \text{ from } (a)\\ \equiv 10\ O_1 + 2x_5 + 4x_4 - 3x_3 + 4x_2 + x_1' \pm e\sqrt{46} \text{ from } (b)\\ \equiv 10\ O_1 + 2x_5 + 4x_4 - 3x_3 + 3x_2 + x_1'' \pm e\sqrt{39} \text{ from } (c)\end{cases}$$

which equations give the ascending series; and it is important to note, that if the probable error of the observations be alike, there is a disadvantage in using any comparison but (*a*), and that even if (*b*) and (*c*) be observed as checks, they should not be used in computing, as they will lower the weight of O_{10}, on the accuracy of which we are dependent for continuing the upward series; thus the mean value of O_{10} from (*a*) and (*c*) will be

$$O_{10} \equiv 10\ O_1 + \tfrac{1}{2}\left(4x_5 + 4x_4 - 5x_3 + 6x_2 + x_1 + x_1''\right) \pm e\sqrt{\tfrac{143}{4}}$$

and if the series (*b*) had been involved the loss of probable accuracy would have been greater.

Next as to descending or decreasing series from W_{10}.

1*st.* Descending through (*a*)

$$O_5 \equiv \tfrac{5}{10} O_{10} + \frac{x_2 - x_1}{2} \pm e\sqrt{\tfrac{50}{10}}$$

$$O_4 \equiv \tfrac{4}{10} O_{10} + \tfrac{1}{10}\left(2x_5 + 4x_4 - 2x_3 - 2x_2 - 4x_1\right) \pm e\sqrt{\tfrac{44}{10}}$$

$$O_3 \equiv \tfrac{3}{10} O_{10} + \tfrac{1}{10}\left(4x_5 - 2x_4 - 4x_3 + x_2 - 3x_1\right) \pm e\sqrt{\tfrac{46}{10}}$$

$$O_2 \equiv \tfrac{2}{10} O_{10} - \tfrac{1}{10}\left(4x_5 - 2x_4 + 6x_3 - 4x_2 + 2x_1\right) \pm e\sqrt{\tfrac{76}{10}}$$

$$O_1 \equiv \tfrac{1}{10} O_{10} - \tfrac{1}{10}\left(2x_5 + 4x_4 - 2x_3 + 3x_2 + x_1\right) \pm e\sqrt{\tfrac{34}{10}}.$$

Again descending through (*b*)

$$O_5 \equiv \tfrac{5}{10} O_{10} + \tfrac{1}{2}\left(x_3 - x_1'\right) \pm e\sqrt{\tfrac{50}{10}}$$

$$O_4 \equiv \tfrac{4}{10} O_{10} + \tfrac{1}{10}\left(2x_5 + 4x_4 + 2x_3 - 6x_2 - 4x_1'\right) \pm e\sqrt{\tfrac{76}{10}}$$

$$O_3 \equiv \tfrac{3}{10} O_{10} + \tfrac{1}{10}\left(4x_5 - 2x_4 - x_3 - 2x_2 - 3x_1'\right) \pm e\sqrt{\tfrac{34}{10}}$$

$$O_2 \equiv \tfrac{2}{10} O_{10} - \tfrac{1}{10}\left(4x_5 - 2x_4 + 4x_3 - 2x_2 + 2x_1'\right) \pm e\sqrt{\tfrac{38}{10}}$$

$$O_1 \equiv \tfrac{1}{10} O_{10} - \tfrac{1}{10}\left(2x_5 + 4x_4 - 3x_3 + 4x_2 + x_1'\right) \pm e\sqrt{\tfrac{46}{10}}.$$

Also descending through (*c*)

$$O_5 \equiv \tfrac{5}{10} O_{10} + \frac{x_3 + x_2 - x_1''}{2} \pm e\sqrt{\tfrac{75}{10}}$$

$$O_4 \equiv \tfrac{4}{10} O_{10} + \tfrac{1}{10}\left(2x_5 + 4x_4 + 2x_3 - 2x_2 - 4x_1''\right) \pm e\sqrt{\tfrac{32}{10}}$$

$$O_3 \equiv \tfrac{3}{10} O_{10} + \tfrac{1}{10}\left(4x_5 - 2x_4 - x_3 + x_2 - 3x_1''\right) \pm e\sqrt{\tfrac{23}{10}}$$

$$O_2 \equiv \tfrac{2}{10} O_{10} - \tfrac{1}{10}\left(4x_5 - 2x_4 + 4x_3 - 4x_2 + 2x_1''\right) \pm e\sqrt{\tfrac{56}{10}}$$

$$O_1 \equiv \tfrac{1}{10} O_{10} - \tfrac{1}{10}\left(2x_5 + 4x_4 - 3x_3 + 3x_2 + x_1''\right) \pm e\sqrt{\tfrac{39}{10}}.$$

If we were to be guided here by the same consideration as before, we should absolutely prefer the use of series (*a*) alone, but it is easy to see, that as the probable error of O_1 involves only $\frac{1}{10}$ of that of O_{10}; the determination of its weight will be almost entirely dependent on the error generated in the comparisons of the group* of the series, and not on that derived from the starting weight: this renders the choice less important.

As a matter of fact I have worked both through (*a*) and (*b*) taking the mean result and in this case.

$$O_5 \equiv \tfrac{5}{10} O_{10} + \tfrac{1}{4}(x_3 + x_2 + x_1 + x_1') \pm e \sqrt{\tfrac{25}{10}}$$
$$O_4 \equiv \tfrac{4}{10} O_{10} + \tfrac{1}{20}(4x_5 + 8x_4 - 8x_2 - 4x_1 - 4x_1') \pm e \sqrt{\tfrac{44}{10}}$$
$$O_3 \equiv \tfrac{3}{10} O_{10} + \tfrac{1}{20}(8x_5 - 4x_4 - 5x_3 - x_2 - 3x_1 - 3x_1') \pm e \sqrt{\tfrac{31}{10}}$$
$$O_2 \equiv \tfrac{2}{10} O_{10} - \tfrac{1}{20}(8x_5 - 4x_4 + 10x_3 - 6x_2 + 2x_1 + 2x_1') \pm e \sqrt{\tfrac{52}{10}}$$
$$O_1 \equiv \tfrac{1}{10} O_{10} - \tfrac{1}{20}(4x_5 + 8x_4 - 5x_3 + 7x_2 + x_1 + x_1') \pm e \sqrt{\tfrac{39}{10}}.$$

My choice was a matter of accident, but it turns out that the sum of the squares of the probable errors of all the deduced weights is less than for any one of the single series.

The other system of weights, which I have in this paper slightly to deal with, is what I shall call the "*English grain system.*" In it the weights interpolated between 10 and 1 are 6, 3 and 2. Thus starting from either end of the decad there are four weights to be derived; but among these weights alone, only three equations can be obtained.

$$\begin{aligned} P_{10} &= P_6 + P_3 + P_1 + x_1 \\ P_6 &= P_3 + P_2 + P_1 + x_2 \\ P_3 &= P_2 + P_1 \qquad\;\; + x_3. \end{aligned}$$

To make a definite resect the best plan is to use a second P_1 called P_1': $P._1 + P._3 + P._1$ from the next lower decad height be used but the equations would not be independent for the separate decads.

$$P_2 = P_1 + P_1' + x_4 \text{ and } P_1 = P_1' + x_5$$

and we now have 5 equations to determine 5 quantities, and the result is definite. Of course by substituting P'_1 for P_1, we can get 3 more equations like the first three, but the labour would be increased, and the result would still be definite, though slightly more accurate, especially as regards the spare weight P_1'.

From the equations we have; *in ascending (increasing weights)*

$$P_1' = P_1 - x_5 \pm e.$$
$$P_2 = 2 P_1 - x_5 + x_4 \pm e \sqrt{2}$$
$$P_3 = 3 P_1 - x_5 + x_4 + x_3 \pm e \sqrt{3}$$

* I use the term *decad* to include the weights from 0·1 to 1, or from 1 to 10, &c., the last being ten times the first; and a *group* of equations consists of those connecting the weights of a *decad.*

$$P_6 = 6\,P_1 - 2x_5 + 2x_4 + x_3 + x_2 \pm e\sqrt{10}$$
$$P_{10} = 10\,P_1 - 3x_5 + 3x_4 + 2x_3 + x_2 + x_1 \pm e\sqrt{24}.$$

While *descending*, we have

$$P_6 = \tfrac{6}{10}\,P_{10} - \tfrac{1}{10}\,(2x_5 - 2x_4 + 2x_3 - 4x_2 + 6x_1) \pm e\sqrt{\tfrac{84}{10}}$$
$$P_3 = \tfrac{3}{10}\,P_{10} - \tfrac{1}{10}\,(x_5 - x_4 - 4x_3 + 3x_2 + 3x_1) \pm e\sqrt{\tfrac{36}{10}}$$
$$P_2 = \tfrac{2}{10}\,P_{10} - \tfrac{1}{10}\,(4x_5 - 4x_4 + 4x_3 + 2x_2 + 2x_1) \pm e\sqrt{\tfrac{56}{10}}$$
$$P_1 = \tfrac{1}{10}\,P_{10} + \tfrac{1}{10}\,(3x_5 - 3x_4 - 2x_3 - x_2 - x_1) \pm e\sqrt{\tfrac{24}{10}}$$
$$P_1' = \tfrac{1}{10}\,P_{10} - \tfrac{1}{10}\,(7x_5 + 3x_4 + 2x_3 + x_2 + x_1) \pm e\sqrt{\tfrac{64}{10}}.$$

Section VII.

I now proceed to the determination of the actual values of the weights below O_1, and of the P set, in commercial grains. The equations have all been determined in terms of the rider R_1, in the balance Oertling No. 1, and they are given in this way. Of course the whole of the computations were made with this unknown factor, but it has been determined (see page 56) and the value has been substituted in the results to save repetition. The differences between the two determinations of the constant term in each equation are given, and from them is derived a probable error of one equation. I had intended that the observations in each decad should be separately valued, but when that is done the results are so nearly alike that it seems unnecessary to adhere to this. The mode of determining the probable error of each weight is the subject of the next section, but the values are given in this.

Value of Weights of W set below W_1 with Balance Oertling No. 1.

I have here the following equations:

O_1	$\equiv O._5$	$+ O._4$	$+ O._1$	$-0{\cdot}213325\ R_1$	Difference	=	2600
O_1	$\equiv O._5$	$+ O._3$	$+ O._2$	$-0{\cdot}238825$,,	,,	=	1450
$O._5$	$\equiv O._4$	$+ O._1$		$-0{\cdot}001800$,,	,,	=	350
$O._5$	$\equiv O._3$	$+ O._2$		$-0{\cdot}124325$,,	,,	=	500
$O._4$	$\equiv O._3$	$+ O._1$		$-0{\cdot}002913$,,	,,	=	825
$O._3$	$\equiv O._2$	$+ O._1$		$-0{\cdot}011113$,,	,,	=	275

$O._1$	$\equiv O._{05}$	$+ O._{04}$	$+ O._{01}$	$-0{\cdot}033200\ R_1$	Difference	=	200
$O._1$	$\equiv O._{05}$	$+ O._{03}$	$+ O._{02}$	$-0{\cdot}042213$,,	,,	=	2925
$O._{05}$	$\equiv O._{04}$	$+ O._{01}$		$-0{\cdot}020938$,,	,,	=	475
$O._{05}$	$\equiv O._{03}$	$+ O._{02}$		$-0{\cdot}032138$,,	,,	=	1475
$O._{04}$	$\equiv O._{03}$	$+ O._{01}$		$-0{\cdot}030838$,,	,,	=	775
$O._{03}$	$\equiv O._{02}$	$+ O._{01}$		$-0{\cdot}035763$,,	,,	=	475

$O._{01} \equiv O._{005} + O._{004} + O._{001}$	$-0.012263\ R_1$	Difference =	425
$O._{01} \equiv O._{005} + O._{003} + O._{002}$	-0.021500 ,,	,, =	150
$O._{005} \equiv O._{004} + O._{001}$	-0.076963 ,,	,, =	1625
$O._{005} \equiv O._{003} + O._{002}$	-0.015813 ,,	,, =	1725
$O._{004} \equiv O._{003} + O._{001}$	-0.040638 ,,	,, =	675
$O._{003} \equiv O._{002} + O._{001}$	-0.093775 ,,	,, =	100
$O._{025} \equiv O._{02} + O._{005}$	$-0.016100\ R_1$	Difference =	200

From these equations I deduce

	grs.		grs.	
$O._5$	$\equiv 240{\cdot}000300 + 0{\cdot}056006\ R_1$	$\equiv$	240·005927	$p.\ e. =$ 0·000064
$O._4$	$\equiv 192{\cdot}000240 + 0{\cdot}127762$,,	$\equiv$	192·013076	,, 0·000060
$O._3$	$\equiv 144{\cdot}000180 + 0{\cdot}100631$,,	$\equiv$	144·010290	,, 0·000047
$O._2$	$\equiv 96{\cdot}000120 + 0{\cdot}081700$,,	$\equiv$	96·008328	,, 0·000048
$O._1$	$\equiv 48{\cdot}000060 + 0{\cdot}030044$,,	$\equiv$	48·003078	,, 0·000037
$O._{05}$	$\equiv 24{\cdot}000030 + 0{\cdot}020606$,,	$\equiv$	24·002100	,, 0·000033
$O._{04}$	$\equiv 19{\cdot}200024 + 0{\cdot}015988$,,	$\equiv$	19·201630	,, 0·000040
$O._{03}$	$\equiv 14{\cdot}400018 + 0{\cdot}021269$,,	$\equiv$	14·402155	,, 0·000033
$O._{02}$	$\equiv 9{\cdot}600012 + 0{\cdot}031475$,,	$\equiv$	9·603180	,, 0·000042
$O._{01}$	$\equiv 4{\cdot}800006 + 0{\cdot}025537$,,	$\equiv$	4·802574	,, 0·000035
$O._{005}$	$\equiv 2{\cdot}400003 + 0{\cdot}030932$,,	$\equiv$	2·403111	,, 0·000033
$O._{004}$	$\equiv 1{\cdot}920002 + 0{\cdot}065261$,,	$\equiv$	1·926559	,, 0·000040
$O._{003}$	$\equiv 1{\cdot}440002 - 0{\cdot}018011$,,	$\equiv$	1·438193	,, 0·000033
$O._{002}$	$\equiv 0{\cdot}960001 + 0{\cdot}033130$,,	$\equiv$	0·963329	,, 0·000042
$O._{001}$	$\equiv 0{\cdot}480001 + 0{\cdot}042634$,,	$\equiv$	0·484284	,, 0·000035
$O._{025}$	$\equiv 12{\cdot}000015 + 0{\cdot}036101$,,	$\equiv$	12·003642	,, 0·000077

The two largest weights P_{24} and P^*_{24} of the P set are each approximately equal to 24 grains and their sum is of course nearly $= O._1$ but they are of platinum while $O._1$ is of gilt bronze. Small as these are the errors cannot be neglected when accuracy is required. The purpose of the determination being mainly to get the values of the small weights of the P set with accuracy so that they may be used to determine differences, it is enough to correct the value above given of $O._1$ so that the deduced value of $P._{24} + P^*_{24}$ may be the same as if the comparison had been made in standard air. For all ordinary purposes the resulting values of these weights may be used without correction.

I have found that 48 grains of platinum would weigh less in my standard air than under the circumstances of the observation by 0.000063 grains. Also $O_1 \equiv P_{24} + P^*_{24} + 0{\cdot}050238\ R_1$.

grs.

The value of $O._1$ is $\equiv 48{\cdot}000060 + 0{\cdot}030044\ R_1$

$\therefore$ in actual air $P_{24} + P^*_{24} \equiv 48{\cdot}000060 - 0{\cdot}020194\ R_1$

and the correction to standard air is $-\ 0{\cdot}000063$

Hence in standard air $P_{24} + P^*_{24} \equiv 47{\cdot}999997 - 0{\cdot}020194\ R_1$

I shall for convenience write M for 47·999997 grains and place the equations so far as they are necessary to determine the weights down to P_1 in a form suitable for use thus—

		Diff.
$P_{24} + P^*_{24}$	$- M + 0{\cdot}020194\, R_1$	$\equiv 0{\cdot}275\ R_1$
$P_{24} - P^*_{24}$	$- 0{\cdot}006438$ „	$\equiv 0{\cdot}475$ „
$P_{24} - P_{16} - P_6 - P_2$	$- 0{\cdot}012913$ „	$\equiv 0{\cdot}1125$ „
$P^*_{24} - P_{20} - P_3 - P_1$	$+ 0{\cdot}000125$ „	$\equiv 0.650$ „
$P_{20} - P_{16} - P_3 - P_1$	$+ 0{\cdot}010138$ „	$\equiv 0{\cdot}900$ „
$P_{20} - P_{10} - P_6 - P_3 - P_1$	$- 0{\cdot}008500$ „	$\equiv 0{\cdot}25$ „
$P_{16} - P_{10} - P_6$	$- 0.018400$ „	$\equiv 0{\cdot}1700$ „
$P_{10} - P_6 - P_3 - P_1$	$- 0{\cdot}004813$ „	$\equiv 0{\cdot}775$ „
$P_6 - P_3 - P_2 - P_1$	$+ 0{\cdot}018908$ „	$\equiv 0{\cdot}375$ „
$P_3 - P_2 - P_1$	$+ 0{\cdot}015463$ „	$\equiv 0{\cdot}1075$ „
$P_2 - P_1 - P'_1$	$+ 0{\cdot}009388$ „	$\equiv 0{\cdot}1175$ „
$P_1 - P'_1$	$- 0{\cdot}005838$ „	$\equiv 0{\cdot}1275$ „

I have tried various ways of dealing with these equations but, when the probable errors are wanted, the method of least squares is the easiest. I thus get—

	grs.		grs.	
P_{24}	$\equiv 23{\cdot}999999$	$- 0{\cdot}006997\, R_1$	$\equiv 23{\cdot}999296$	*p. e.* = 0·000042
P^*_{24}	$\equiv 23{\cdot}999999$	$- 0{\cdot}003185$ „	$\equiv 23{\cdot}998679$	„ 0·000042
P_{20}	$\equiv 19{\cdot}999999$	$- 0{\cdot}014515$ „	$\equiv 19{\cdot}998541$	„ 0·000050
P_{16}	$\equiv 15{\cdot}999999$	$- 0{\cdot}006007$ „	$\equiv 15{\cdot}999396$	„ 0·000049
P_{10}	$\equiv 9{\cdot}999999$	$- 0{\cdot}009026$ „	$\equiv 9{\cdot}999092$	„ 0·000043
P_6	$\equiv 6{\cdot}000000$	$- 0{\cdot}015531$ „	$\equiv 5{\cdot}998440$	„ 0·000013
P_3	$\equiv 3{\cdot}000000$	$- 0{\cdot}006360$ „	$\equiv 2{\cdot}999361$	„ 0·000035
P_2	$\equiv 2{\cdot}000000$	$+ 0{\cdot}001371$ „	$\equiv 2{\cdot}000137$	„ 0·000050
P_1	$\equiv 1{\cdot}000000$	$+ 0{\cdot}008077$ „	$\equiv 1{\cdot}000811$	„ 0·000039
P'_1	$\equiv 1{\cdot}000000$	$+ 0{\cdot}002461$ „	$\equiv 1{\cdot}000247$	„ 0·000043

			Diff.	
Further	$P_1 \equiv P_{\cdot 6} + P_{\cdot 3} + P_{\cdot 1}$	$+ 0{\cdot}000038\, R_1$	Diff.	$725\ R_1$
	$P_{\cdot 6} \equiv P_{\cdot 3} + P_{\cdot 2} + P_{\cdot 1}$	$+ 0{\cdot}005525$ „	„	0 „
	$P_{\cdot 3} \equiv P_{\cdot 2} + P_{\cdot 1}$	$- 0{\cdot}004675$ „	„	500 „
	$P_{\cdot 2} \equiv P_{\cdot 1} + P'_{\cdot 1}$	$+ 0{\cdot}006963$ „	„	1325 „
	$P_{\cdot 1} \equiv P'_{\cdot 1}$	$+ 0{\cdot}005813$ „	„	525 „

			grs.	
Whence $P_{\cdot 6}$	$\equiv 0{\cdot}600000$	$+ 0{\cdot}002673\, R_1$	$\equiv 0{\cdot}600269$	*p. e.* = 0·000056
$P_{\cdot 3}$	$\equiv 0{\cdot}300000$	$+ 0{\cdot}005647$ „	$\equiv 0{\cdot}300567$	„ 0·000035
$P_{\cdot 2}$	$\equiv 0{\cdot}200000$	$+ 0{\cdot}002832$ „	$\equiv 0{\cdot}200285$	„ 0·000042
$P_{\cdot 1}$	$\equiv 0{\cdot}100000$	$+ 0{\cdot}000842$ „	$\equiv 0{\cdot}100085$	„ 0·000028
$P'_{\cdot 1}$	$\equiv 0{\cdot}100000$	$- 0{\cdot}004971$ „	$\equiv 0{\cdot}099501$	„ 0·000045

By weighing the riders against the nearly equal weight $P._1$ I have

$$R_1 \equiv P._1 + 0{\cdot}003813\,R_2 \qquad \text{Diff.} \quad 425$$
$$R_2 \equiv P._1 + 0{\cdot}000375\,R_1 \qquad ,, \quad 600$$

Substituting successively for the value of R_1, of $P._1$, and of R_2 we get

$$\begin{array}{ll} & \text{grs.} \\ R_1 \equiv & 0.1003814 + 0{\cdot}000847\,R_1 \equiv 0{\cdot}100466 \text{ grs. } p.\ e. = 0{\cdot}000062 \\ R_2 \equiv & 0{\cdot}100000 \quad + 0{\cdot}001217\,R_1 \equiv 0{\cdot}100122 \quad ,, \quad ,, \quad = 0{\cdot}000062 \end{array}$$

$$\begin{array}{lllll} \text{Also—}P._1 & \equiv P._{06} + P._{03} + 0{\cdot}089038\,R_2 & & \text{Diff.} & 825 \\ P._{06} & \equiv P._{03} + P._{02} + 0{\cdot}104750 & ,, & ,, & 1550 \\ P._{03} & \equiv P._{02} \qquad + 0{\cdot}105075 & ,, & ,, & 900 \\ P._{01} & \equiv \qquad 0{\cdot}099438 & ,, & ,, & 137 \end{array}$$

$$\begin{array}{llll} \text{Whence } P._{06} & \equiv \tfrac{2}{3}\,P._1 - 0{\cdot}059467\,R_2 \equiv 0{\cdot}060769 & p.\ e. = & 0{\cdot}000047 \\ P._{03} & \equiv \tfrac{1}{3}\,P._1 - 0{\cdot}029571 \ ,, \equiv 0{\cdot}030400 & ,, & 0{\cdot}000034 \\ P._{02} & \equiv \tfrac{1}{3}\,P._1 - 0{\cdot}134646\,R_1 \equiv 0{\cdot}019881 & ,, & 0{\cdot}000047 \\ P._{01} & \equiv \qquad 0.099438 \ ,, \equiv 0{\cdot}009956 & ,, & 0{\cdot}000056 \end{array}$$

SECTION VIII.—*Determination of the probable errors of the values of the O and P sets.*

In Section VI, I have shown that if the probable error of the constant terms in the equations of a group be known, we can determine the probable errors of the determinations in the group, so far as they depend on it: and we have now to consider what may be taken as the probable error of one determination.

Each coefficient of R is derived in the preceding work from two determinations which rarely agree. The differences are noted in terms of the 6th decimal place of the coefficient. If we were certain that the true values of the constants lay between the determinations, then, calling the difference of the two $2a$, we should have $\frac{\Sigma\, a}{n}$ = the mean of errors and $p.\ e.$ of an equation $= e = 0{\cdot}8454\,\frac{\Sigma\, a}{n}$; but this value is clearly too small; because, if the occurrence of positive and negative errors be equally probable, then there is an even chance that a fourth of the values of $2a$ will be the difference and not the sum of the two actual errors.

I prefer therefore to use the formula

$$\text{mean of errors} = \frac{\Sigma\, v}{\sqrt{m\,(m-1)}}$$: m being the number of complete comparisons

$$\text{and probable error} = 0{\cdot}8454\,\frac{\Sigma\, v}{\sqrt{m\,(m-1)}}$$

applying this to any one determination we shall have its probable error

$$= 0{\cdot}8454\,\frac{2\,a}{\sqrt{2\times1}} = 0{\cdot}8454\,\sqrt{2}a = 1{\cdot}1955\,a$$

Of course this is a very uncertain estimation, but we have a good many such equations, and the mean of the values may I think be taken as the fairest estimate. If then n be the number of equations, I take

$$p.\ e.\ \text{of any one determination is } 1\cdot1955\ \frac{\Sigma\, a}{n}$$

The group of equations determining the P weights would give the probable error from their residuals; but, there being only 12 equations to determine 10 quantities, I do not think this is so satisfactory as the above method; and I have used, for evaluating the errors in them, the weights of the results, deduced as usual, combined with the *p. e.* of an equation derived as above. Assuming that we may neglect the difference between the values of R_1 and R_2 in these differences, we have 41 values of $2\ a$; and it does not seem that there is any marked tendency to decrease with the weights: I therefore take the mean of all and I get

$$\frac{\Sigma\, a}{n} = 463\cdot53\ R \qquad p.\ e. = 554\cdot16\ R = 55\cdot651 = e \text{ of Section VI}$$

$$\text{in which R is taken } 0\cdot100464 = \frac{36\,R_1 + 5\,R_2}{41}$$

Hence e^2 is $3097\cdot0$

The probable error of any determination as of that of $O._{03}$ for instance, depends:—

1st on the amount arising from its own group.

2nd probable error of the value assumed as known: in this case $O._1$

3rd on the probable error of the rider which was employed in taking the difference of weights in the pans.

Lastly O_1 itself has its probable error $0\cdot000115$ grains from the determinations; but there is also a portion dependent on $P._{01}$, which is involved in determining the difference between it and EI, the mean factor of $P._{01}$ being $0\cdot0877$. It is necessary, therefore, to start our evaluations with values of the probable errors of R_1 R_2 and $P._{01}$; and, fortunately, these are readily determined.

Let E be the *p. e.* of $P._1$ from all sources except R_1

e as before the *p. e.* of one determination

ϵ the *p. e.* of R_1

It will be seen from the table of deduction of probable errors that the value of E^2 is $758\cdot2$ and that it involves nothing unknown.

$$\begin{aligned}\text{Hence } (p.\,e.\,R_1)^2 &= \epsilon^2 \\ &= (1\cdot003813)^2\ E^2 + (0\cdot000842)^2\ \epsilon^2 + e^2 \\ &= 764\cdot0 + 0\,0000007\ \epsilon^2 + 3097\cdot0 = 3861\cdot0\end{aligned}$$

$$\therefore \epsilon = 0\cdot000062 = \frac{1}{10^6}\sqrt{3861\cdot0}$$

again $p.\ e.\ R_2 = \sqrt{E^2 + e^2 + 0{\cdot}000375^2\ \epsilon^2} = \frac{1}{10^6}\sqrt{3861{\cdot}0} = 0\ 000062$

$p.\ e.\ P{\cdot}_{01} = \sqrt{e^2 + 0{\cdot}099438^2\ (R_2)^2} = \sqrt{3135{\cdot}2} = 0{\cdot}000056$

Determination of Probable Errors.

	Squares of Probable Errors (unit is 6th decimal place).						Probable error.
	From group.	From preceding groups.	From EI.	From R_1.	From $P{\cdot}_{01}$.	Total.	
O_1	...	...	13225·0	...	24·1	13249·1	0·000 115
$O{\cdot}_5$	774·3	...	3306·2	12·1	6·0	4098·6	64
$O{\cdot}_4$	1362·7	...	2116·0	64·2	3·9	3546·3	60
$O{\cdot}_3$	960·1	...	1190·3	41·1	2·2	2193·7	47
$O{\cdot}_2$	1734·3	...	529·0	25 7	1·0	2290·0	48
$O{\cdot}_1$	1207·8	...	132·3	3·5	0·2	1343·8	37
$O{\cdot}_{05}$	774·3	301·9	33·1	1·6	0·1	1111·0	33
$O{\cdot}_{04}$	1362·7	193·2	21·2	1·0	,,	1578·1	40
$O{\cdot}_{03}$	960·1	108·7	11·9	1·7	,,	1082·4	33
$O{\cdot}_{02}$	1734·3	48·3	5·3	3·8	,,	1791·7	42
$O{\cdot}_{01}$	1207·8	12·1	1·3	2·5	,,	1223·7	35
$O{\cdot}_{005}$	774·3	305·0	0·3	3·7	,,	1083·3	33
$O{\cdot}_{004}$	1362·7	195·2	0·2	16·4	,,	1574·5	40
$O{\cdot}_{003}$	960·1	109·8	0·1	1·2	,,	1071·2	33
$O{\cdot}_{002}$	1734·3	48·8	0·1	4·2	,,	1787·4	42
$O{\cdot}_{001}$	1207·8	12·2	,,	7·0	,,	1227·0	35
$O{\cdot}_{025}$	3097·0	2861·9	8·3	19·8	,,	5987·0	77
P_{24}	1447·5	301·9	33·1	0·2	0·1	1782·8	42
P^*_{24}	1447·5	301·9	33·1	,,	0·1	1782·6	42
P_{20}	2310·6	209·7	22·9	,,	,,	2543·2	50
P_{16}	2229·2	134·2	14·7	0·1	,,	2378·2	49
P_{10}	1806·4	52·4	5·7	,,	,,	1864·5	43
P_6	148·1	18·9	0·9	0·5	,,	168·4	13
P_3	1245·2	4·7	0·5	0·1	,,	1250·5	35
P_2	2541·5	2·1	0·2	,,	,,	2543·8	50
P_1	1490 5	0·5	,,	0·3	,,	1491·3	39
P'_1	1836·0	0·5	,,	,,	,,	1836·5	43
$P{\cdot}_6$	2601·5	536·9	,,	,,	,,	3138·4	56
$P{\cdot}_3$	1114·9	134·2	,,	,,	,,	1249·1	35
$P{\cdot}_2$	1734·6	59·6	,,	,,	,,	1794 2	42
$P{\cdot}_1$	743·3	14·9	,,	,,	,,	758·2	28
$P'{\cdot}_1$	1982·1	14·9	,,	,,	,,	1997·0	45

$$\text{Also } p.\,e.\,P._{06} = \frac{1}{10^6}\sqrt{2064{\cdot}6+169{\cdot}5+13{\cdot}6} = \frac{1}{10^6}\sqrt{2247{\cdot}7} = 0{\cdot}000047$$

$$p.\,e.\,P._{03} = \frac{1}{10^6}\sqrt{1032{\cdot}3\;+\;84{\cdot}2+13{\cdot}3} = \frac{1}{10^6}\sqrt{1129{\cdot}8} = 0{\cdot}000034$$

$$p.\,e.\,P._{02} = \frac{1}{10^6}\sqrt{2064{\cdot}6\;+\;84{\cdot}2+70{\cdot}0} = \frac{1}{10^6}\sqrt{2218{\cdot}8} = 0{\cdot}000047$$

Section IX.—*Determinations of the Weights O_2 to O_{10} and also Prinsep's Bronze Troy Pound.*

The comparisons of the weights from O_2 to O_{10} have been made with the balance Oertling No. 2. Three complete comparisons were made in each case, and the weight $P._{03}$ has been always used for valuing the scale. I have deduced the following equations of condition:—

grs.

$$O_3 \equiv O_2+O_1 \qquad -0{\cdot}37200\,P._{03} \equiv O_2+O_1 \quad 0{\cdot}000000-0{\cdot}37200\,P._{03}$$
$$O_4 \equiv O_3+O_1+P._{06}+0{\cdot}74542\,P._{03} \equiv O_3+O_1+0{\cdot}060769+0{\cdot}74542\,P._{03}$$
$$O_5 \equiv O_3+O_2+P._1 \;+0{\cdot}37867\,P._{03} \equiv O_3+O_2+0{\cdot}100085+0{\cdot}37867\,P._{03}$$
$$\equiv O_4+O_1+P._{02}+0{\cdot}60467\,P._{03} \equiv O_4+O_1+0{\cdot}019881+0{\cdot}60467\,P._{03}$$
$$O_{10} \equiv O_5+O_4+O_1-P._1-P._{06}+0{\cdot}45742\,P._{03} \equiv O_5+O_4+O_1- 0{\cdot}160854+0{\cdot}45742\,P._{03}$$

Whence I deduce by the Formulæ in Sec. VI.

$$O_2 \equiv 2\,O_1+P._{06}+P._{02}-P._1+0{\cdot}97142\,P._{03} \equiv 960{\cdot}011294 \text{ grs.}$$
$$p.\,e. = 0{\cdot}000757 \text{ ,,}$$
$$O_3 \equiv 3\,O_1+P._{06}+P._{02}-P._1+0{\cdot}59942 \text{ ,,} \equiv 1440{\cdot}000584 \text{ ,,}$$
$$p.\,e. = 0{\cdot}000900 \text{ ,,}$$
$$O_4 \equiv 4\,O_1+2\,P._{06}+P._{02}-P._1+1{\cdot}34484 \text{ ,,} \equiv 1920{\cdot}084613 \text{ ,,}$$
$$p.\,e. = 0{\cdot}001194 \text{ ,,}$$
$$O_5 \equiv 5\,O_1+2\,P._{06}+2\,P._{02}-P._1+1{\cdot}94951 \text{ ,,} \equiv 2400{\cdot}123435 \text{ ,,}$$
$$p.\,e. = 0{\cdot}001438 \text{ ,,}$$
$$O_{10} \equiv 10\,O_1+3\,P._{06}+3\,P._{02}-3\,P._1+3{\cdot}75167 \text{ ,,} \equiv 4800{\cdot}061736 \text{ ,,}$$
$$p.\,e. = 0{\cdot}002795 \text{ ,,}$$

In the last Section, I have given a general formula for finding a probable error of observation. In this case, I have $\Sigma\,(o) = 3941{\cdot}2\,\frac{P._{03}}{10^5}$, whence the probable error of one equation of condition will be

$$= 0{\cdot}8454\cdot\frac{3941{\cdot}2}{\sqrt{3{\cdot}2}}\cdot\frac{P._{03}}{10^5} = 0{\cdot}000413{\cdot}5$$

The probable error of each determination of a weight depends—

1st, on its error derived from O_1 of which it is nearly a multiple,

2nd, on the error derived through the weights of the P set used to nearly counterbalance, 3rd, on the error due to the fraction of $P._{03}$ which is involved in its determination,

4th, on the error generated in the weighings of the series.

The following Table shows the error from each source separately.

Weights.	Squares of Probable Errors from					Probable Error × 10^6.
	O_1	Equil. Weights.	$P._{03}$	Weighments of Series.	Total.	
O_2	52900	5225	1179	514116	573420	757
O_3	119025	5225	419	685488	810187	900
O_4	211600	11968	2259	1199600	1425427	1194
O_5	330625	18624	4747	1713720	2067716	1438
O_{10}	1922500	47022	47581	5826648	7813751	2795

In making these calculations, I have neglected to attend to the fact that the P weights used have a common origin ; the sum of the squares of the probable errors given in the Table at the end of Section VIII is taken, and here (as will be seen by turning back) the error from their common origin $O._1$ is unfelt, but this is not always the case.

Among the weights in the Assay Office is a bronze Standard Troy Pound in a wooden case, on which case is stamped $\left\{\begin{array}{c}\text{J. FIELD}\\ \text{Fecit}\end{array}\right\}$, and in ink is written

J. Prinsep.
Std. 1 lb

On the weight itself is engraved—

British Troy Pound.
= 5760 *grains.*
Royal Mint.

The surface of the weight is thinly oxidized, but it seems to be quite uninjured. I some time ago compared it, as well as I could, with the weights of the Gilt Troy set belonging to the Assay Office, which were supplied many years ago, and which were made by Bates in 1824. No record of any previous comparisons of these exists. The conclusion I came to was, that Prinsep's Troy Pound was about a mean of all the Gilt Pounds, the latter weights having sensible errors. I have then thought it worth while to determine the value of the Prinsep's Pound, and I find—

Prinsep's Pound $\equiv O_{10} + O_2 + P_1 + P._{01} - 0{\cdot}487\ P._{03}$

$\equiv 5760{\cdot}148354$ grains,

from a single complete comparison.

To find the probable error of this we must substitute in the above equation the symbolic values of $O_{10} + O_2$ and thus we have—

Prinsep's Pound $\equiv 12\,O_1 + P._{01} + 4\,P._{02} + 4\,P._{06} - 3\,P._{1} + 4\,23606\,P._{03}$
from which the probable error will (when the errors generated in determining O_2 and O_{10}, and also in the single comparison of this weight are allowed for)

$$= \frac{1}{10^6}\sqrt{8878998} = 0{\cdot}002890$$

and we may consider Prinsep's Pound $\equiv 5760{\cdot}148 \pm 0\,003$ grains.

Section X.—*Considerations as to the Weights which should be made use of in a series.*

The only generally used decimal system of weights, is the metric, which is so largely diffused. In it the weights between W_1 and W_{10} are W_5, W_2 in duplicate, and W_1. When the system was adopted in England permissively, the intermediate weights chosen were W_5 W_3 and W_2. The other series in use, are those I have described before as the Bullion, and the English Grain Series. In making a series of weights of tolahs for the use of the Indian mints, I have therefore a choice; and it is worth considering which series is the best.

Commercially, the fewer weights required to make any weighment, the better. I think, too, that commercially it is undesirable to have duplicate weights, and of course none should be superfluous. In the strict *French Metric* system there are 3 weights required to weigh 9 and 8, while two are wanted for 7, 6, and 3, and the 2 is in duplicate; and in the *English modification* there are 3 weights wanted for 9 only, while 8, 7, 6, and 4 require two each, and there is no duplicate: I think then that the English modification is preferable to the original system.

In our *English Bullion* system there are never 3 weights wanted for any purpose; and 9, 8, 7, and 6 require two weights. But there are more weights than are wanted, there being 5 weights in each decad instead of 4.

In the *English Grain* system there are never 3 weights wanted; 9, 8, 7, 5. and 4 require two each, there are no duplicates, and none superfluous. I think then that the English Grain system is the best for commercial purposes.

Scientifically, the best system is that of which the values can be most accurately deduced from the standard Prototype. It is worthy of note, that neither of the Metric systems, nor the English Grain system, admit of the weights of a decad being completely determined without a second unit in each decad.

This is not an unmixed disadvantage. The 1, 10, &c., being necessary for this purpose only, and not used in common, may be kept separately, and referred to for verifications whenever desired, and by such use the errors of the weights of any decad, can be determined with comparatively little labour and without its being necessary to refer back to a primary weight. Thus, checking becomes much more manageable, and, by such a plan as I have adopted in dealing with the P set, one of the duplicates is far more accurately determined than the other, and can be laid aside for reference; the accuracy of the second being ordinarily sufficient.

The English Bullion system, as we have seen, contains the means of determining the values of all the weights without duplicates, and it is possible to have one weight practically unused, if we consent to make either 8 or 9 by three weights; this reference weight, however, is not so convenient for use as in the other cases.

The English Grain system has this advantage over all the others, that any weight from 1 to 10 requires at most two weights to make it. It has the disadvantage that 6 is not the half of ten, but, on the other hand, 3 is the half of 6; and I do not see the great gain of this relation, unless it be admitted that the system of division should be binary. In France, it was proposed that each multiple of a unit by ten, and each division by ten, should be a new unit. Some slight gain might have come if this had become a thoroughly practical procedure; but, in fact, one rarely hears of any but the kilogramme, gramme, and milligramme, and so of the other numbers of the series. I think, then, that the advantage of being able to have a single weight for half a hectogramme, &c. is dearly purchased, if there be a disadvantage in the determinations; and, in deciding on a system of weight, it is necessary to consider the probable errors of these determinations.

In each of these proposed systems, 5 comparisons, giving 5 equations, are enough to connect all the weights in a decad. If this number be alone used, then the probable errors of W_{10} derived from W_1 will be

English Grain System	......	$e\sqrt{24}$	
„ Bullion	....	$e\sqrt{34}$	if the best equation be taken.
„ Metric	....	$e\sqrt{38}$	
Original Metric	....	$e\sqrt{26}$	

In this respect the English Grain system seems best, and the Modified Metric System the worst. The Original Metric system is nearly as good as the English Grain system, and it is possibly better if a good deal more labour be given to each; but I think—when it is considered that weighing by the English Grain system requires only two weights in each decad, and that the standard system should coincide if possible with that in use—the palm will be assigned to the Grain system.

I think, too, that those who have gone with me so far, will feel as strongly as myself the great gain of a "large primary unit." It has always been considered necessary to have the primary unit very indestructible, and no doubt this is a very important point: the lead was taken in France, where the Normal Kilogramme was made of platinum; platinum was again used in England for the Standard Pound, and now standards of reference are made of a Platinum-iridium alloy. The cost of the mere metal is very heavy (a kilogramme is at present worth £60 for mere material), and the use of such a metal for large weights is of course out of the question. It seems to me doubtful whether equal accuracy could not be obtained by employing a large weight of gilt or nickelized bronze; from which copies could be made with far greater accuracy than they could be separately deduced from the small primary. It is possibly too late to change the material of Primary Standards now, but at all events the standard of Commercial Weight should be a large mass of gilt bronze.

Acting on these principles, I have nearly made a set of weights from 1000 tolahs to 0·001 tolah from these bullion weights. There will be several copies of the largest, carefully compared, some of which I trust Government will allow me to distribute. The individual weights are on what I have called the English Grain system: that is, there are—

1000 tolahs.	100 tolahs.	10 tolahs.	1· tolahs.	0·10 tolahs.	0·010 tolahs.
600 „	60 „	6 „	0·6 „	0·06 „	0·006 „
300 „	30 „	3 „	0·3 „	0·03 „	0·003 „
200 „	20 „	2 „	0·2 „	0·02 „	0·002 „
100 „	10 „	1 „	0·1 „	0·01 „	0·001 „

The final adjustments and deductions have yet to be made; but after what I have said, there will be little new in this. I have been very greatly assisted by Mr. Durham, Senior Assistant in the Assay Office, who has superintended all of the gilding; and to whom I owe devices which will allow the gilt weights to be made true almost to the accuracy of a single comparison by substitution.

Table I.

Logarithms for calculating the Weight of the Air adapted to Fahrenheit's Thermometer.

This Table gives 10 + the logarithm of the ratio which the weight of air at the temperature named and at Calcutta bears to that of the same volume of water when at its maximum density, the logarithm of the height of the barometer.

If B be the reading of the barometer reduced to freezing point; the temperature and V the elasticity of the vapour in the air

$$\text{then log sq. of air} = A_t + \log (B - 0{\cdot}238\ V).$$

The value of A_t at sea-level in latitude 45° can be got from these numbers by adding 0·000785·7 to each and thence the value for any other place.

Temp.	A_t.	$\Delta^{(1)} A_t$.	Temp.	A_t.	$\Delta^{(1)} A_t$.	Temp.	A_t.	$\Delta^{(1)} A_t$.
30°	5·6366164	8848	55°	5·6150200	8419	80°	5·5944469	8030
1	6357316	8830	6	6141781	8402	1	5936439	8015
32	6348486	8812	7	6133379	8387	2	5928424	8000
3	6339674	8794	8	6124992	8371	3	5920423	7985
4	6330880	8776	9	6116621	8354	4	5912438	7971
35	5·6322104	8759	60	5·6108267	8338	85	5·5904467	7957
6	6313345	8741	1	6099929	8323	6	5896510	7942
7	6304604	8724	2	6091606	8306	7	5888568	7927
8	6295380	8705	3	6083300	8291	8	5880641	7913
9	6287175	8689	4	6075009	8275	9	5872728	7899
40	5·6278486	8671	65	5·6066734	8258	90	5·5864829	7884
1	6269815	8654	6	6058476	8244	1	5856945	7870
2	6261161	8637	7	6050232	8227	2	5849075	7856
3	6252524	8619	8	6042005	8212	3	5841219	7841
4	6243905	8603	9	6033793	8197	4	5833378	7828
45	5·6235302	8585	70	5·6025596	8181	95	5·5825550	7813
6	6226717	8569	1	6017415	8166	6	5817737	7799
7	6218148	8552	2	6009249	8151	7	5809938	7785
8	6209596	8535	3	6001098	8135	8	5802153	7772
9	6201061	8518	4	5992963	8120	9	5794381	7757
50	5·6192543	8502	75	5·5984843	8105	100	5·5786624	
1	6184041	8485	6	5976738	8090			
2	6175556	8468	7	5968468	8074			
3	6167088	8452	8	5960514	8060			
4	6158636	8436	9	5952514	8045			

Table II.

Logarithm of the Ratio of the Density of Water to its Maximum Density for each degree of Fahrenheit's Thermometer.

This Table is founded on that given at page 66 &c. of the Report of the Warden of the Standards for 1871-72. Certain values of the Table there given, were taken and the constants found to express them in a series of the form A $(t - n_1)^2$ + B $(t - n_2)^3$, and, these having then been suitably modified to change the scale of the thermometer from Centigrade to Fahrenheit, the present Table was computed.

Temp.	Log. Ratio.	$\Delta^{(1)}$ R.	Temp.	Log. Ratio.	$\Delta^{(1)}$ R.	Temp.	Log. Ratio.	$\Delta^{(1)}$ R.
30°			55°	0·0002400	+302	80°	0·0014313	639
1			6	0002702	318	1	0·0014952	650
2	0·0000546	—143	7	0003020	335	2	0015602	659
3	0000404	—121	8	0003355	350	3	0016261	670
4	0000283	— 99	9	0003705	367	4	0016931	679
35	0·0000184	— 78	60	0·0004072	381	85	0·0017610	688
6	0000106	— 56	1	0004453	397	6	0018298	698
7	0000050	— 35	2	0004850	412	7	0018996	706
8	0000015	— 15	3	0005262	426	8	0018702	715
9	0000000	+ 06	4	0005688	441	9	0020417	723
40	0·0000006	+ 27	65	0·0006129	455	90	0·0021440	732
1	0000033	47	6	0006584	469	1	0021872	739
2	0000080	66	7	0007053	483	2	0022611	747
3	0000146	86	8	0007536	497	3	0023358	754
4	0000232	105	9	0008033	509	4	0024112	762
45	0·0000337	124	70	0·0008542	523	95	0·0024874	768
6	0000461	144	1	0009065	535	6	0025642	775
7	0000605	162	2	0009600	548	7	0026417	782
8	0000767	180	3	0010148	560	8	0027199	787
9	0000947	198	4	0010708	572	9	0027986	794
50	0·0001145	216	75	0·0011280	584	100	0·0028780	
1	0001361	234	6	0011864	596			
2	0001595	251	7	0012460	607			
3	0001846	269	8	0013067	617			
4	0002115	285	9	0013684	629			

TABLE III.

Logarithms for facilitating the Calculation of the Cubical Expansion of Metals.

Log. $(1 + EM_t.)$

	G = M Gold — 339·14	S = M Silver — 441·41	P = M Platinum — 208·32	B = M Baily's metal — 394·98.	Br = M Brass — 398·27
1	0 000010598	0·000013794	0 000006510	0 000012343	0·000012446
2	21196	27588	13020	24686	24892
3	31794	41382	19530	37029	37338
4	42392	55176	26040	49372	49784
5	52990	68970	32550	61715	62230
6	63588	82764	39060	74058	74676
7	74186	96558	45570	86401	87122
8	84784	110352	52080	98744	99568
9	95382	124046	58590	111087	112014

This table is founded on the supposition that up to 100° of Fahrenheit's Thermometer; log expansion for $n° = n \times$ log expansion for 1°; which is true sufficiently. The linear expansions of Gold and Silver have been taken from Vol. I of Professor Miller's Chemistry; the others from the paper in the 'Philosophical Transactions' on Standard Weights.

The argument of this Table is to be T — 32°; or T itself can be taken if the number at the head of the column be applied.

Thus for brass at 85·35° we have

Br 50°	0·000622·30	or Br 80°	0·000995·68
3	37·34	5	62·23
0·3	3·73	·3	3·73
0·05	0·62	·05	0·62
		Const.	— 398·27
	0·000663·99		0·000663·99

Type Comparison I.

May 24th, 1879. Oertling, No. 1.

Comparisons of EI with O_1.

Weight on left side.	Weight on right side.	Scale Readings. Low.	Scale Readings. High.	Deduced Mean.	Remarks.
EI	$O_1 + 1{\cdot}2\frac{R_2}{10}$	5·7 6·1	9·5 9·2	7·54	Commenced at $6^{\rm h}{\cdot}48^{\rm m}$ A. M. A. Bar. $29{\cdot}60^{\rm in.}$ Temp. $85{\cdot}0^\circ$ F. Dry Bulb $85{\cdot}9^\circ$. Wet Bulb $81{\cdot}0^\circ$.
EI + $P._{10}$	Do.	13·6 13·7	18·0 17·8	15·81	
$O_1 + P._{01}$	EI + $1{\cdot}2\frac{R_2}{10}$	13·1 13·4	17·4 17·2	15·21	
O_1	Do.	3·4 3·8	10·7 10·3	6.95	
Do.	Do.	3·4 3·8	10·0 9·6	6·60	
$O_1 + P._{01}$	Do.	13.3 13·6	16·6 16·3	15·03	
EI + $P._{01}$	$O_1 + 1{\cdot}2\frac{R_2}{10}$	12·8 13·3	18·9 18·5	15·99	
EI	Do.	3·0 3·6	11.9 11·4	7·61	Bar. $29{\cdot}61^{\rm in.}$ Temp. $86{\cdot}0^\circ$ F. Dry Bulb. $85{\cdot}4^\circ$ Wet Bulb $80{\cdot}1^\circ$. Ended at $7^{\rm h}{\cdot}33^{\rm m}$ A. M.

$$\text{Hence EI} \doteq O_1 + 1{\cdot}2\frac{R_2}{10} - \frac{2{\cdot}46}{8{\cdot}27}P._{01} \doteq O_1 + 1{\cdot}2\frac{R_2}{10} - 0{\cdot}297\ P._{01}.$$

$$\text{EI} \doteq O_1 - 1{\cdot}2\frac{R_2}{10} + \frac{3{\cdot}05}{8{\cdot}26}P._{01} \doteq O_1 - 1{\cdot}2\frac{R_2}{10} + 0{\cdot}369\ P._{01}.$$

$$\text{EI} \doteq O_1 - 1{\cdot}2\frac{R_2}{10} + \frac{3{\cdot}40}{8{\cdot}43}P._{01} \doteq O_1 - 1{\cdot}2\frac{R_2}{10} + 0{\cdot}404\ P._{01}.$$

$$\text{EI} \doteq O_1 + 1{\cdot}2\frac{R_2}{10} - \frac{2{\cdot}39}{8{\cdot}38}P._{01} \doteq O_1 + 1.2\frac{R_2}{10} - 0{\cdot}285\ P._{01}.$$

$$\therefore 4\ \text{EI} \equiv 4\ O_1 + 0{\cdot}191\ P._{01} : \text{or EI} \equiv O_1 + 0{\cdot}04775\ P._{01}.$$

Note.—In the original the succession of observations has been distinguished, but want of space rendered it necessary to give this up.

TYPE COMPARISON II.

June 5th, 1879. Oertling No. 1.

Comparisons of O_1 with $O._5 + O_4 + O._1 = S$.

Weight on left side.	Weight on right side.	Scale Readings. Low.	Scale Readings. High.	Deduced Mean.	Remarks.
$O_1 + 5\frac{R_1}{10}$	$S_1 + 4{\cdot}2\frac{R_2}{10}$	6·3 6·6	10·2 10·0	8.34	
$O_1 + 6\frac{R_1}{10}$	Do.	13·0 13·4	19·0 18·6	15·90	
$S + 0{\cdot}6\frac{R_1}{10}$	$O_1 + 4{\cdot}2\frac{R_2}{10}$	3·0 3·3	10·6 10·3	6·88	
$S + 1{\cdot}6\frac{R_1}{10}$	Do.	11·0 11·4	17·6 17·2	14·40	
Do.	Do.	9·9 10·4	19.4 18·8	14·49	
$S + 0{\cdot}6\frac{R_1}{10}$	Do.	4·1 4·4	9·7 9·4	6·98	
$O_1 + 6\frac{R_1}{10}$	$S + 4{\cdot}2\frac{R_2}{10}$	12·8 13·1	17·9 17·4	15·40	
$O_1 + 5\frac{R_1}{10}$	Do.	6·0 6·2	9·9 9·6	7·99	

Hence $O_1 \triangleq S + 4{\cdot}2\frac{R_2}{10} - \left(5{\cdot}0 + \frac{1{\cdot}66}{7{\cdot}56}\right)\frac{R_1}{10} \triangleq S + 4{\cdot}2\frac{R_2}{10} - 0{\cdot}5226\,R_1.$

$$O_1 \triangleq S - 4{\cdot}2\frac{R_2}{10} + \left(0{\cdot}6 + \frac{3{\cdot}12}{7{\cdot}52}\right)\frac{R_1}{10} \triangleq S - 4.2\frac{R_2}{10} + 0{\cdot}1015\,R_1.$$

$$O_1 \triangleq S - 4{\cdot}2\frac{R_2}{10} + \left(0{\cdot}6 + \frac{3{\cdot}02}{7{\cdot}51}\right)\frac{R_1}{10} \triangleq S - 4{\cdot}2\frac{R_2}{10} + 0\ .002\,R_1.$$

$$O_1 \triangleq S + 4{\cdot}2\frac{R_2}{10} - \left(5{\cdot}0 + \frac{2{\cdot}01}{7{\cdot}41}\right)\frac{R_1}{10} \triangleq S + 4{\cdot}2\frac{R_2}{10} - 0{\cdot}5272\,R_1.$$

$\therefore 4\,O_1 \equiv 4\,S - 0{\cdot}8481\,R_1$ or $O_1 \equiv O._5 + O._4 + O._1 - 0.212025\,R_1$

TYPE COMPARISON III.

October 22nd, 1879. Oertling No 2.

Comparisons of O_5 with $O_1 + O_4 + P._{02} = S$.

Weight on left side.	Weight on right side.	Scale Readings. Low.	Scale Readings. High.	Deduced Mean.	Remarks.
S	O_5	9 5 9·9	14·1 13·8	11·91	
$S + P._{03}$	Do.	15·0 15·5	22·7 22·2	18·73	
O_5	$S + P._{03}$	12 0 12·2	14·6 14·3	13·34	
Do.	S	16·3 16·7	23·0 22·6	19·55	
Do.	Do.	16·1 16·6	23·3 22·8	19·58	
Do.	$S + P._{03}$	12·2 12·4	14·1 14·0	13·21	
$S + P._{03}$	O_5	15·4 15·7	21·0 20·7	18·13	
S	Do.	10·8 10·9	13·0 12·8	11·91	

$$\text{Hence } O_5 \doteq S + \frac{3·09}{6·82} P._{03} \doteq S + 0·453\ P._{03}.$$

$$O_5 \doteq S + \frac{4·55}{6·21} P._{03} \doteq S + 0·732\ P._{02}.$$

$$O_5 \doteq S + \frac{4·58}{6·21} P._{03} \doteq S + 0·737\ P._{03}.$$

$$O_5 \doteq S + \frac{3·09}{6·22} P._{03} \doteq S + 0·497\ P._{03}.$$

$$\therefore 4\ O_5 \equiv 4\ S + 2·419\ P._{03} \text{ and } O_5 \equiv S + 0·60475\ P._{03}.$$

$$\equiv O_1 + O_4 + P._{02} + 0·61475 P._{03}.$$

P. S. *June 29th*, 1880.—After the earlier part of this paper was drafted, I learnt that M. St. Claire Deville had proposed to make standards of the Commercial Kilogram in a new manner. The metal is to be the Platinum-iridium alloy so as to secure hardness and indestructibility, but, in order that the density may be nearly that of brass, it is to be hollow, the parts are to be soldered together by fusion so as to enclose a constant mass of air, which, of course, will be included in the weighings. This plan has been adopted by the International Commission for making the European Metric Standards, and will no doubt be a great improvement on the old Commercial Standard of France, which is made of brass. The volume of these weights is to be 125 cubic centimetres, so that the density will be 8·0; which is a little lower than that of good sound weights of brass, and materially lower that that of gilt bronze; while it is greater than that of iron.

Certainly, the visible Commercial unit, to which reference can be made, appears preferable to the imaginary unit of England. Such a weight would vary in Calcutta with respect to the scientific unit to the extent of about 11 milligrams, and it would be needless to take notice (for commercial purposes) of the much smaller variations with respect to such weight as may be compared with it.

VI.—*On the High Atmospheric Pressure of* 1876-78 *in Asia and Australia, in relation to the Sun-spot Cycle.—By* Henry F. Blanford, *Met. Rep. to the Govt. of India.*

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(With Plate I.)

The three years 1876, 1877, and 1878, more especially the two former, were characterized by a deficiency of rainfall in one or many parts of India, and by a more general and very persistent excess of atmospheric pressure. With but slight and local interruptions, from August (in some parts of India from May) 1876 to August (in some cases only to May) 1878, over the whole of the Indian area, the barometer ranged above the average of many years. Nor was this excess of pressure restricted to the land. The register of Port Blair at the Andaman Islands, and that of Nancowry at the Nicobars, shew that, at these insular stations, the excessive pressure was of greater duration and more persistent and intense than at any continental station at or near the sea-level; indeed, with one striking exception, more intense than at any other station in the entire region. At these islands, the pressure rose above the average in May 1876; and, from that time to August 1878 inclusive, the mean pressure of every month was from ·004″ to ·071″ in excess of the average; derived, in the case of Port Blair